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Clausius Mosotti Equation:

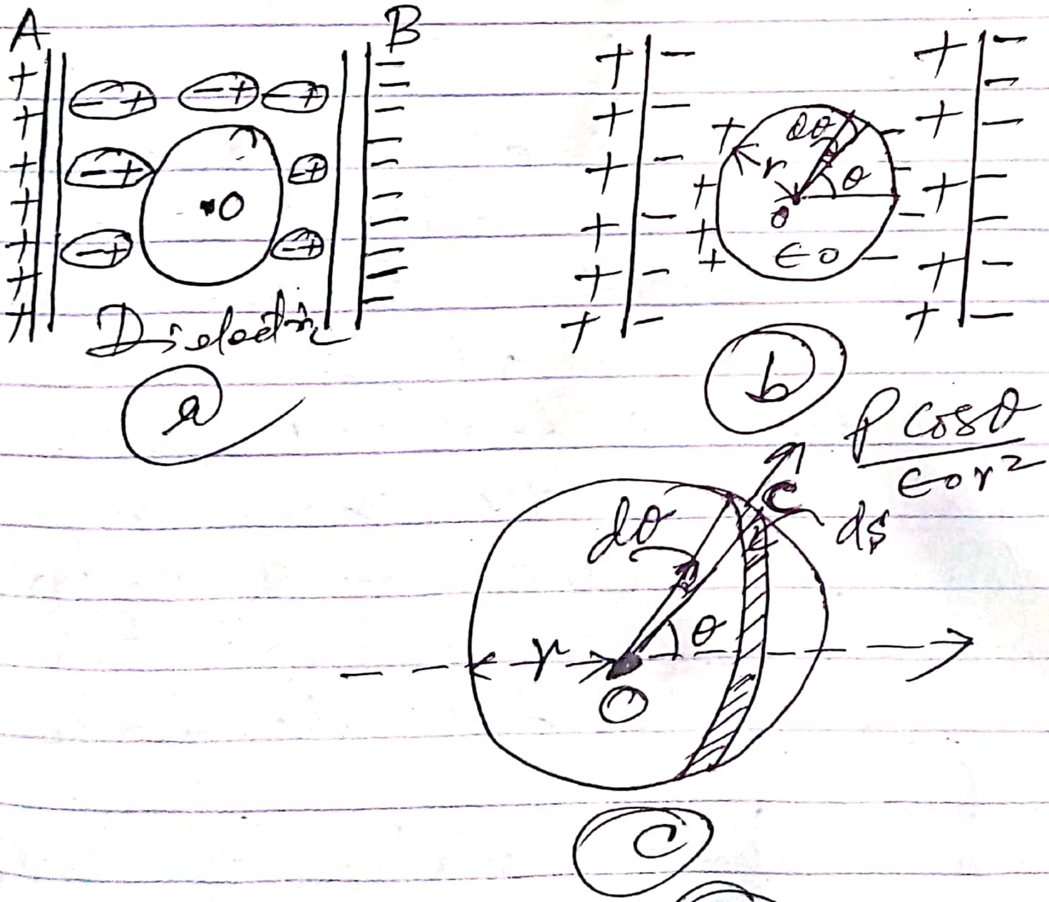

Fig. 4

As from the definition of electric polarisation; when a dielectric is placed in an electric field, then due to induction, its polar molecules orientate themselves along the direction of the applied ~~magnetic~~ field. Each molecule is an electric dipole possessing an electric moment. The average induced electric dipole moment per unit volume of the dielectric is called dielectric polarisation, and is denoted by P. The amount of polarisation is proportional to the applied electric field i.e., $P \propto E$, or $P = \alpha E$; where α is the constant of proportionality depending on the dielectric, and is called molecular polarisability of the dielectric. The electric field responsible for polarising a molecule of the dielectric, is called the molecular field and is denoted by E_m. It is produced by all external sources and by all polarised molecules in the dielectric with the exception of one molecule at the point under consideration. E_m is different from E; the latter is related to the force on a test charge.

(4) **Calculation of the molecular field:**

Suppose a thin dielectric sample has been polarised in a uniform field

... parallel plates, [Fig. 4 (a)]. Let the non-polar dielectric be uniformly polarised.

Let us imagine a spherical cavity in the dielectric. The part of the dielectric external to the cavity may be replaced by a system of bound charges as shown in Fig. 4 (B). Let ϵ_r = Relative permittivity or Dielectric constant of the dielectric and ϵ_0 is the permittivity of the free space.

Consider the sphere to have two surface densities of charge $+\sigma$ and $-\sigma$ which coincide when there is no external field. Let r be the radius of the cavity surrounding the free space. The electric field E_m in the free space in the cavity, or the force acting on a unit charge at the centre O will be due to two components

(1) E_1 due to charge σ_r per unit area on the parallel conducting plates A and B on either side of the dielectric. If this dielectric were a free space $E_1 = \frac{\sigma_r}{\epsilon_0}$; and (2) E_2 due to the polarised dielectric surrounding the cavity, but E_2 has two components,

(a) E_p; the force due to the bound charges

of σ_p per unit area on the dielectric facing the plates, and
(b) E_s, produced by the charge on the surface of the cavity due to polarisation P.

$$\text{But } E_p = -\frac{\sigma_p}{4\pi\epsilon_0} = -\frac{P}{\epsilon_0} \cdot (\text{S.I. units})$$

(E_p being −ve because E is +ve)

$E_s = ??$

Let $d\sigma$ be a surface element of the sphere at C with polar coordinates r, θ. The normal component of P at C is $P\cos\theta$, charge per unit area over $d\sigma$ [Fig. 4(c)]

$\therefore$ Force on unit charge at O $= \dfrac{P\cos\theta\, d\sigma}{4\pi\epsilon_0 r^2}$

directed along OC.

The component of this force along E

$$= \frac{P\cos\theta\, ds}{4\pi\epsilon_0 r^2} \cdot \cos\theta;$$

and the component in the perpendicular direction

$$= \frac{P\cos\theta\, ds}{4\pi\epsilon_0 r^2} \cdot \sin\theta.$$

If the element $d\sigma$ is rotated through $360°$ about the axis through O but parallel to E, the area dS of the ring shaped element described on the surface is given by

$$dS = 2\pi r \sin\theta \cdot r\,d\theta.$$

The component of the force at O perpendicular to E due to this ring is zero because such components are symmetrically distributed around the axis.

$\therefore$ Component of force along $E = \dfrac{2\pi r^2 \sin\theta\, d\theta\, P\cos^2\theta}{4\pi\epsilon_0 r^2}$.

$\therefore$ Total force E_s per unit charge at O due to surface charge on the cavity

$$= \int_0^{\pi/2} \frac{P\cos^2\theta}{2\epsilon_0 r^2} \cdot 2\pi r^2 \sin\theta\, d\theta$$

$$= \frac{P}{2\epsilon_0}\int_0^{\pi/2} \cos^2\theta \sin\theta\, d\theta = \frac{P}{3\epsilon_0} \quad \text{--- (1)}$$

Therefore total force on unit charge at O due to all the forces considered above is

$$E_m = E_1 + E_2 = E_1 + E_p + E_s$$

$$= \frac{\sigma_r}{\epsilon_0} - \frac{P}{\epsilon_0} + \frac{P}{3\epsilon_0}.$$

Since we know that

$$D = \sigma_r = \epsilon_0 E + P$$

~~E_m~~ where E = resultant intensity

$$\therefore E = \frac{\sigma r}{\epsilon_0} - \frac{P}{\epsilon_0}$$

$$\therefore E_m = E + \frac{P}{3\epsilon_0} \quad \text{---} \quad (2)$$

which gives the value of the intensity of molecular field.

Let us consider the case of an isotropic gaseous non-polar dielectric and assume that each atom is a sphere. The field causing the polarisation of such an atom may be considered as given by equation (1). The electric moment of the atom can be put $= \alpha E_m$, where α is a constant for the atom. If there are n atoms per unit volume of the gas

or, $P = n\alpha E_m$;

$$\therefore n\alpha = \frac{P}{E_m}$$

$\therefore$ Polarisation per unit volume is

$$P = n\alpha E_m = n\alpha\left(E + \frac{P}{3\epsilon_0}\right). \quad \text{---} \quad (3)$$

If ϵ = absolute permittivity of gaseous dielectric, the displacement $\underline{\underline{D}}$ is given by

$$D = K\epsilon_0 E = \epsilon_0 E + P = \epsilon E$$

where ϵ = absolute permittivity

or, $P = E(\epsilon_0 K - \epsilon_0)$

or, $E = \dfrac{P}{(\epsilon_0 K - \epsilon_0)}$.

Substitute in (2)

$$\therefore E_m = \frac{P}{\epsilon_0 K - \epsilon_0} + \frac{P}{3\epsilon_0} = \frac{P(2\epsilon_0 + \epsilon_0 K)}{3\epsilon_0(\epsilon_0 K - \epsilon_0)}$$

or $P = \dfrac{3\epsilon_0(\epsilon_0 K - \epsilon_0)E_m}{(\epsilon_0 K + 2\epsilon_0)}$

$$\therefore \frac{P}{E_m} = \frac{3\epsilon_0(\epsilon_0 K - \epsilon_0)}{(\epsilon_0 K + 2\epsilon_0)}$$

But from (3),

$P = n\alpha E_m$ or $\alpha = \dfrac{P}{nE_m}$

$$\therefore \alpha = \frac{3\epsilon_0(\epsilon_0 K - \epsilon_0)}{n(\epsilon_0 K + 2\epsilon_0)}$$

or, $\alpha = \dfrac{3\epsilon_0}{n}\left[\dfrac{K-1}{K+2}\right] = \dfrac{3\epsilon_0(K-1)}{n(K+2)}$.

$$\therefore \frac{K-1}{K+2} = \frac{n\alpha}{3} \quad \text{---} \quad (4)$$

Eqn. (4) is known as Clausius Mossotti equation. Since n is proportional to the density of the gas, it follows that if α is constant for a particular gas, then $\frac{K-1}{K+2}$ is proportional to the density of the gas, which has been verified experimentally.